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Dropping or Marking: A Review and Evaluation of Existing Fluid-Flow Approximation Models

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Abstract. Fluid-flow approximation is an approach to modeling and evaluating the performance of vast computer networks. Due to varying traffic and performance of transmission protocols reacting to traffic overloads, computer networks are in a permanent transient state. The fluid-flow method's main advantage is its ability to analyse these transient states. The article reviews and organises several versions of this approach, indicating a few errors. The main reason for these errors is confusion or lack of distinction between the two versions of the Internet Protocol - when the queue of packets at a node is too long, they may be destroyed or only marked as redundant. The paper compares and evaluates these fluid-flow approximation models with mild and aggressive settings of RED parameters. The authors build a software system with hitherto unprecedented capabilities regarding the size of the networks to be analysed and with innovative way of organising the calculations. The paper shows how large differences imprecise assumptions can introduce in quantitative results.

Key words: fluid-flow approximation; transient states; TCP/IP models; AQM; modelling

1. INTRODUCTION

Queuing theory, e.g. [1], is used to study computer networks. Queueing models have the form of a network of service stations where customers are served and queued when waiting for their service. The service stations represent computer network nodes, and customers represent packets or blocks transmitted across the network. The service time is the time needed to send a packet to the output link leading to the next node of the packet's itinerary. The queueing delay depends on the current state of the network and is a stochastic variable. Its determination is vital to evaluate the transmission quality.

Analytical models of network protocols allow us, based on mathematical equations, to better understand the performance of the protocols in various network topologies and various work conditions. Their use makes the analysis much less time- and resource-consuming than in the case of discrete-event simulation while maintaining sufficient accuracy of the results. The models allow us to fully understand how the protocols work and find inefficiencies, especially those not anticipated by the original designers. They also indicate possible areas for optimising the protocols, often specifying the potential increase in performance or usability. Moreover, they significantly facilitate the verification of multi-aspect concepts and enable the comparison of relevant fragments or entire rules of individual protocols at the same level.

This way, queueing theory provides mathematical and numerical methods for predicting the behaviour of queues. It enables analysing the behaviour of devices and networks under various load conditions and examining the impact of their parameters on such values as throughput, delay and packet losses. It supports network optimisation, which is crucial for its efficiency, availability and quality of services. It is beneficial when transient state models are applied, i.e. queues change in time, reacting on time-variable flows. Three mathematical tools are usually applied: Markov chains, diffusion approximation and fluid flow approximation.

2. CLASSIC FLUID-FLOW APPROXIMATION MODEL

In numerical analysis, a single router is rarely the object of interest. The most common way to evaluate the performance of a network is to represent it as a set of N interconnected nodes and a set of K connections passing through those nodes.

The first parameter that strongly depends on the network structure is RTT (round trip time) – the average time, after which the sender receives confirmation of receipt of the packet. RTT of the i-th flow denoted as R_i in eq. (1) and the following ones, depends on the total queuing delay and the total propagation delay (Tp_i), which is equal to the sum of the propagations of all links along the route. The total queuing delay is defined as the sum of the individual queuing delays (the quotient of the instantaneous queue length q_j and the service intensity C_j) of the N_i routers along the i-th flow path from the source to the destination

$$R_i(q(t)) = \sum_{j=1}^{N_i} \frac{q_j(t)}{C_j} + Tp_i \,, \quad N_i \in N. \tag{1}$$

The size W of the congestion window (the number of packets that could be sent without waiting for the recepient's acknowledgement) defines the dynamics of TCP flow. In the classic version of the TCP protocol [2], the window increases by one with each subsequent acknowledgement (every RTT time on average), so with the speed $1/R_i(t)$, and it is halved with each packet loss. In the equation below, the loss intensity is represented by the term $\frac{W_i(t-\tau)}{R_i(t-\tau)}P_i(t-\tau)$, i.e. the product of the flow intensity and loss probability P_i of an individual packet. The sender performs the changes of W with a certain

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delay τ:

$$\frac{dW_i(t)}{dt} = \frac{1}{R_i(t)} - \frac{W_i(t)}{2} \cdot \frac{W_i(t-\tau)}{R_i(t-\tau)} \cdot P_i(t-\tau). \quad (2)$$

Queue length changes in the network model, eq. (3), are based on the intensity of the input and output streams. Flows (1..K) are grouped into K_s classes corresponding to connections. Let K_v be the number of flows entering a given node v in the network. The sum of these flows' throughputs $W_i(t)/R_i(t)$ defines the input stream. The output stream has intensity C_v given by the speed of service at the node

$$\frac{dq_v(t)}{dt} = \sum_{i=1}^{K_v} \frac{W_i(t)}{R_i(q_i(t))} - \mathscr{H}(q_v(t) > 0) \cdot C_v. \quad (3)$$

where $\mathscr{H}(\cdot)$ is Heaviside step function.

The primary goal of all Internet protocols is to maintain high throughput and low latency along the transmission path. To meet this demand, active queue management in routers was introduced [3, 4]. There are two methods of notifying the sender of congestion. In the standard approach, network devices (including popular CISCO routers) use the packet *drop* strategy, [5]. In an alternative method, i.e. ECN (*Explicit Congestion Notification*), routers (instead of discarding the packet) *mark* it by setting a particular bit in the message header, if the sender's transport protocol allows it. Packets of a flow are deleted (or marked) not only when there is no place to store them in the node buffers but also when the intensity of the flow increases dangerously. The standard approach is defined by RED - Random Early Deletion (or Detection) algorithm [6]. The algorithm randomly chooses the packets to be deleted or marked with a certain probability. A single intervention is enough to signal a problem to the source of the particular flow.

In the classical fluid-flow approximation model, the RED algorithm is modelled. It monitors the moving average queue length, eq. (5), to determine the packet rejection probability at the router, eq. (4).

$$p_v(x_v(t)) = \begin{cases} 0, & 0 \leqslant x_v < t_{min_v} \\ \frac{x_v(t) - t_{min_v}}{t_{max_v} - t_{min_v}} \cdot p_{max_v}, & t_{min_v} \leqslant x_v \leqslant t_{max_v}, \\ 1, & t_{max_v} < x_v \leqslant B_v \end{cases} \quad (4)$$

$$x_{v_n}(t) = \alpha_v \cdot q_{v_n}(t) + (1-\alpha_v) \cdot x_{v_{n-1}}(t). \quad (5)$$

3. DISCUSSION OF EXISTING MODELS BASED ON FLUID-FLOW APPROXIMATION

The classic fluid approximation model, described in [7] and Appendix A of the [8] report, does not distinguish between the method of notifying the source of congestion, using the concepts of packet dropping (D) and packet marking (M) interchangeably. However, if the packet is only marked, it is not physically removed from the node's queue, i.e. despite exceeding the maximum RED threshold, the queue may still grow. In the case of packet dropping, when the probability is equal to one, the queue values do not increase. The classic model leads to incorrect numerical results, Table 1.Z1. The extension introduced in [9] specifies the notification model as ECN marking, correctly defining the current queue equation, Table 1.Z1. Unfortunately, it also presumes equality between marking and dropping a packet: *"Each link $l \in E$ is served at C_l bps rate. In addition, associated with each link is an AQM policy, characterised by a probability discarding/marking function $p_l(t)$, which may depend on link state such as queue length"*.

The final report made by the authors of the fluid model, [8], is also only sometimes clear. The central part of the report, Table 1.Z2, does not specify the general notification method. Still, compared to the steady-state model, it emphasises that *"the approach applies equally if the active management policy queue marks packets rather than drops them"*. It, therefore, assumes no significant differences in modelling both behaviours. The report presents the equation determining the full use of an overloaded node in a steady state, considering the dropping mechanism. Still, it does not directly define the type of method. The authors of the model in Appendix B of the report and [11] assume packet dropping as the primary method, although the presented equation concerns the marking method, Table 1.Z3, which they finally point to in the numerical experiments section. Appendix E of the report and [10], in turn, clearly defines the formula for the q queue, Table 1.Z1, as relating to ECN marking. However, it does not present an equation representing packet dropping. Similarly, the work [15] does not consider the method of dropping and marking packets while providing the equation for marking, Table 1.Z5.

The lack of distinction between methods also appears in [12] and Appendix F of the report. In the theoretical part, the authors put the equation indicating packet dropping, Table 1.Z4, however in the content, they use both concepts interchangeably, and in the numerical results, they emphasise the use of marking. An analogous equation is presented in the work [13]. The authors assume the given queue model with packet dropping. A similar model was presented in [14], however, specifying directly the use of the packet dropping method for a queue handling aggregated flows and packets. The alternative method (ECN) is not discussed in both cases.

A model with packet dropping is also presented in [16]. It divides the traffic into flows connected directly to a given node and connections from other routers, Table 1.Z6. Only the authors of the publication [17] analytically point out the difference between queue models working according to the dropping and marking algorithms. Still, they consider a different approximation method (*mean-field approximation*) with the number of flows approaching infinity.

Fluid-flow approximation model is still being used and developed. The paper [18] proposes a novel TCP-AQM mechanism to extend TCP packet loss cycles and reduce flow completion times for time-sensitive applications in data centers using a modification of classical fluid-flow model. The authors of [19] analyzes the TCP/AQM transmission mechanism in multi-node networks using diffusion approximation and compares its performance with the fluid-flow approach. The study [20] develops and validates an adaptive dynamic surface controller to manage congestion in wireless TCP networks, which is based on an extended fluid-flow approximation. This paper [21] develops an adaptive congestion control algorithm using fuzzy logic and backstepping techniques to address challenges

ID	Reference	Actual Queue Length Definition	Method
Z1	[7], [8].A	$\frac{dq(t)}{dt} = -\mathscr{H}(q(t)>0)\cdot C + \sum_{i=1}^{K} \frac{W_i(t)}{R_i(t)}\cdot K_i$	D/M
	[9]		D/M
	[10], [8].E		M
Z2	[8]	$\frac{dq(t)}{dt} = -\mathscr{H}(q(t)>0)\cdot C + \sum_{i=1}^{K} \frac{W_i(t)}{R_i(t)}$	D/M
Z3	[11], [8].B	$\frac{dq(t)}{dt} = -\mathscr{H}(q(t)>0)\cdot C + \sum_{i=1}^{K} A_i(t)\cdot K_i$	D/M
Z4	[12], [8].F	$\frac{dq(t)}{dt} = -\mathscr{H}(q(t)>0)\cdot C + (1-p(t))\sum_{i=1}^{K} A_i(t)\cdot K_i$	D/M
	[13]		D
	[14]		
Z5	[15]	$\frac{dq(t)}{dt} = -C + u(t) + \frac{W_i(t)}{R_i(t)}\cdot K_i$	D/M
Z6	[16]	$\frac{dq(t)}{dt} = -\mathscr{H}(q(t)>0)\cdot C + (1-p(t))(\sum_{i=1}^{K_1} \frac{W_i(t)}{R_i(t)} + \sum_{j=1}^{K_2} A_j(t))$	D

Table 1. A comparison of equations defining the instantenous queue length at a node in the literature, where D stands for packet dropping and M for packet marking

in multi-bottleneck TCP/AQM networks with nonlinearity and disturbances using modified fluid-flow model. In [22] an enhanced Proportional Integral (PI) controller is proposed for congestion control in computer networks. Similarly, the [23] proposes a robust Fractional Order PI (FOPI) controller to enhance congestion control in TCP networks. In both cases, the modeling was performed using the fluid-flow approximation.

4. NUMERICAL COMPARISON OF PACKET DROPPING AND MARKING MODELS

The principle of packet dropping (D) assumes that when the maximum RED threshold is exceeded, the node does not accept any new packets into its buffer. Selecting a packet to be dropped depends on the probability determined by the RED algorithm, $p_v(t)$. For zero loss, all input traffic is accepted into the node. An increase in load results in the selective dropping of packets and reducing the input stream by a loss percentage equal to the value of the probability $p_v(t)$. Reaching congested node will trigger dropping all newly arrived packets. Therefore, it is reasonable to use the probability: $1-p_v(t)$ as a factor conditioning and characterising the size of new packets acceptance into the queue from the input stream:

$$\frac{dq_v(t)}{dt} = -\mathscr{H}(q_v(t)>0)\cdot C_v + \sum_{i=1}^{K_v}\left(\frac{W_i(t)}{R_i(t)}\cdot K_i\cdot(1-p_v(t))\right) \quad (6)$$

In the case of the marking method (M), selected packets are allowed into the queue, regardless of the value of the probability $p_v(t)$. Even when the node is overloaded ($p_v(t)=1$), all input traffic is sent to the buffer. Therefore, it is correct to adopt the classic formula for this case:

$$\frac{dq_v(t)}{dt} = -\mathscr{H}(q_v(t)>0)\cdot C_v + \sum_{i=1}^{K_v}\left(\frac{W_i(t)}{R_i(t)}\cdot K_i\right) \quad (7)$$

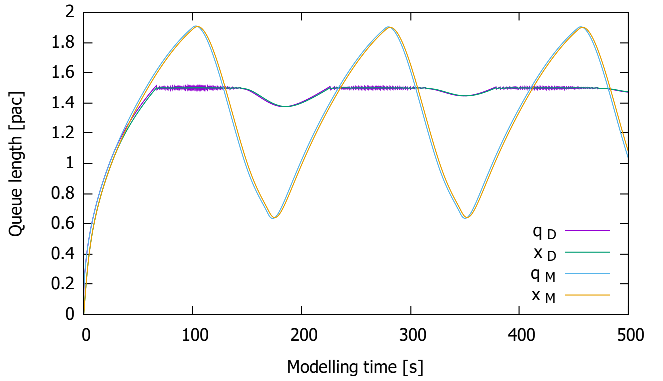
Numerical analyses were carried out to confirm the above statements. The first scenario involved compiling the results for a single node, working according to the RED algorithm, firstly in the dropping mode (D) and then in the marking (M) mode. The router assumed a service intensity of 0.075 pac/s and could accept a maximum of 15 packets. The minimum and maximum RED thresholds were set at 1 and 1.5 packages, respectively. The weight parameter was 0.05. The probability of a proactive response was set to every 10th packet ($p_{max_v}=0.1$), and then the aggressiveness was increased for every second packet ($p_{max_v}=0.5$). A single flow with a total link propagation of 300 ms was routed through the node.

Figures 1 and 2 illustrate the behaviour of individual parameters of the model with dropping and marking of packets. Figures 1a and 1b indicate the correct behaviour of both models concerning queue length. For the model D, the queue values do not increase when the maximum RED threshold is exceeded, which is the case with the M model. According to the algorithm logic, in the case of packet dropping, using both mild and aggressive settings significantly reduces the occurrence of oscillations in the queue value, visible in the M model. Using the M model leads to long periods of total node overload while reaching the probability value at the maximum level, fig. 1c and fig. 1d. A different situation occurs in the model implementing real losses (D) without permanent overload. For the mild mode, there are individual moments when all packets are dropped, but these are short-lived states, fig. 1c. However, with an aggressive setting, fig. 1d, the mechanism does not go beyond the preventive phase.

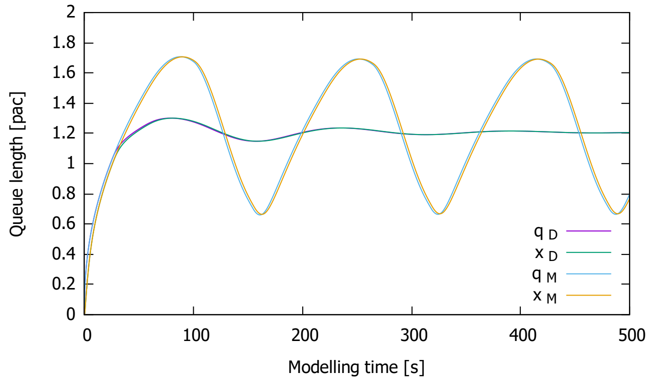
Introducing overload (M model) causes a significant reduction in the average size of the transmission window, which can be observed in the figure for the marking algorithm, fig. 1e and fig. 1f. Not only do the oscillations form, but the overload window drops to the initial values, which is a highly unfavourable phenomenon. In model D, short-term overloads lead to much smaller decreases in the window value, which tends to be in a steady state, fig. 1e and fig. 1f.

Changing the method of informing the source about the node state in the examined case influenced the flow capacity. Model D has over 16% (fig. 2a and fig. 2c) and over 21% (fig. 2b and fig. 2d) higher average throughput, which is related to the higher average size of the congestion window of the D model compared to the M model. The average value of RTT times is at a similar level in both models, fig. 2e and fig. 2f, while the amplitude of the value changes is significantly different.

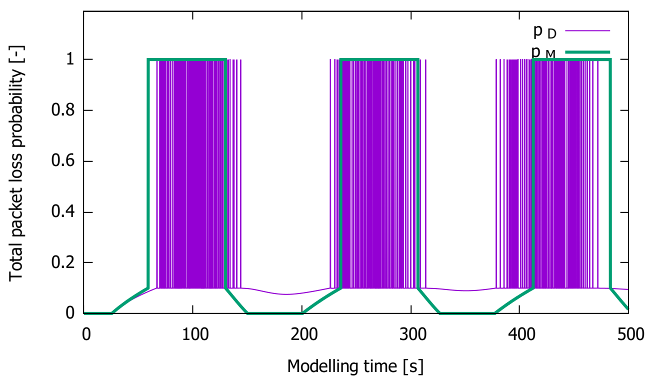
The second scenario assumed verification of the behaviour



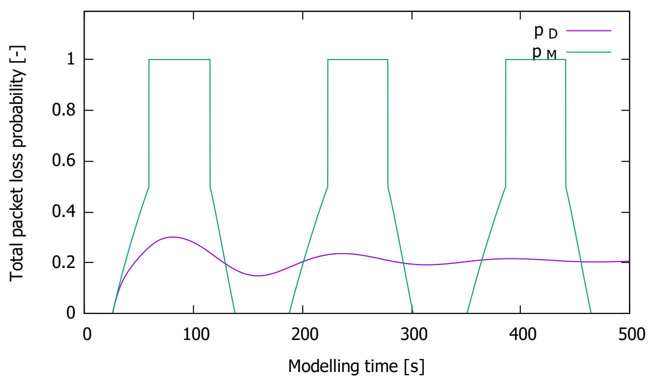
(a) Comparison of the instantaneous q_v and the average x_v queue length for a mild setting of the parameter $p_{max_v} = 0.1$



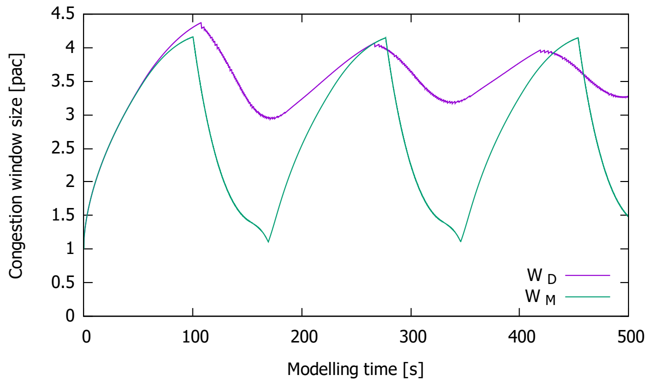
(b) Comparison of the instantaneous q_v and the average x_v queue length for an aggressive setting of the parameter $p_{max_v} = 0.5$



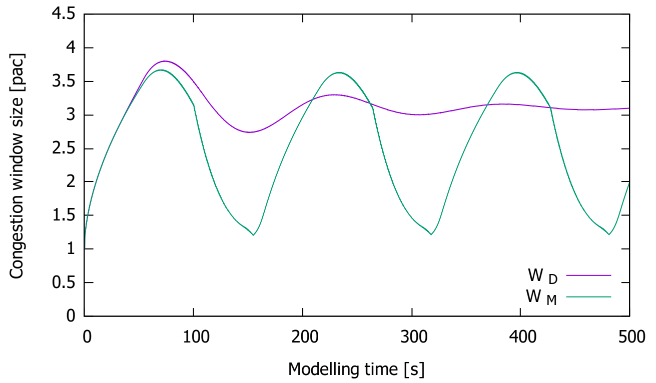
(c) Comparison of the router's proactive probability for a mild setting of the parameter $p_{max_v} = 0.1$



(d) Comparison of the router's proactive probability for an aggresive setting of the parameter $p_{max_v} = 0.5$

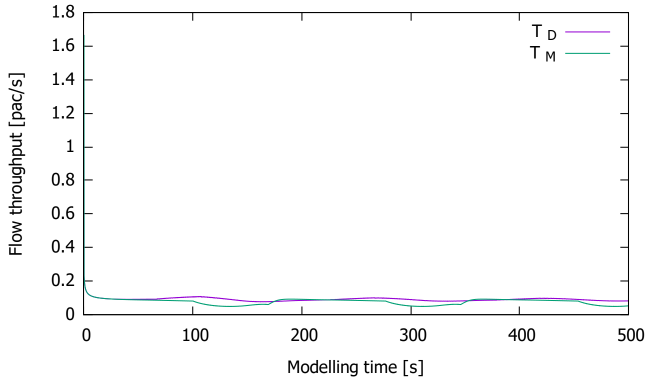


(e) Comparison of the flow congestion window size for a mild setting of the parameter $p_{max_v} = 0.1$

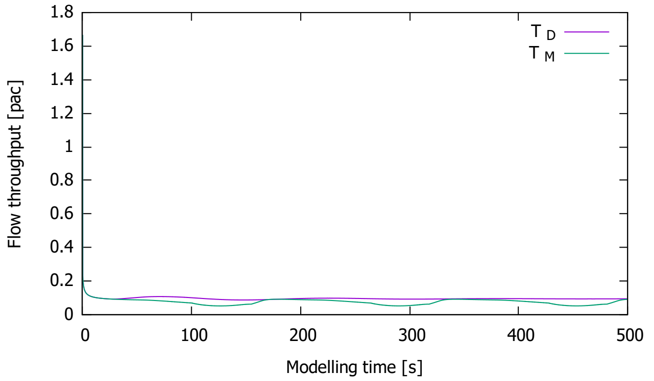


(f) Comparison of the flow congestion window size for an aggressive setting of the parameter $p_{max_v} = 0.5$

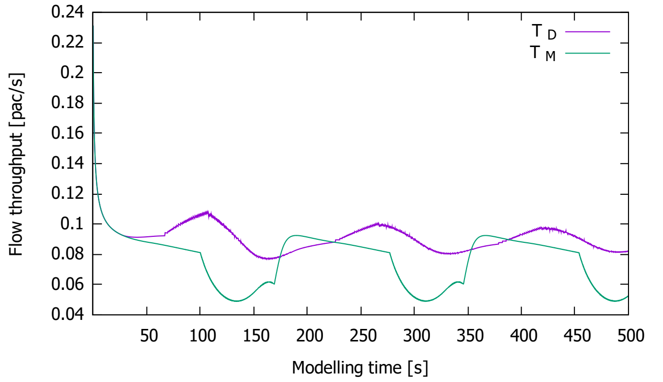
Fig. 1. Comaparison of numerical results of queue lengths, RED probabilities and congestion window sizes for a single flow and a single node operating according to the packet dropping (D) or packet marking (M) algorithm for a mild and aggressive setting of the p_{max_v} parameter. Individual categories of i flows are marked with colours.



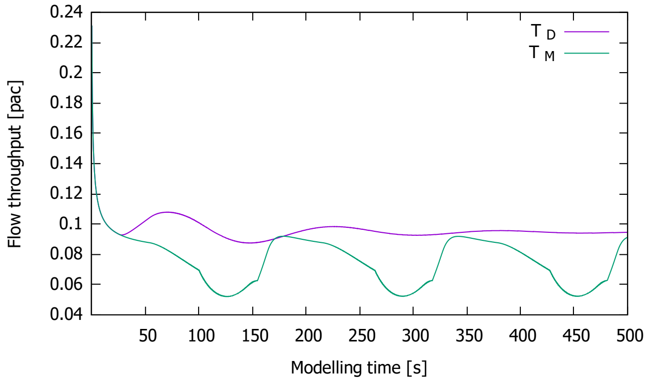
(a) Comparison of flow throughput for a mild setting of the parameter $p_{max} = 0.1$



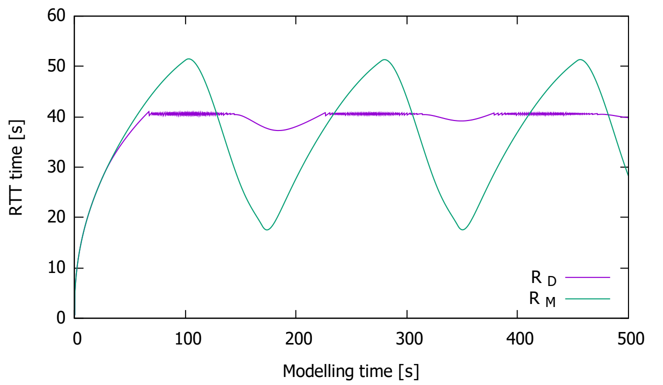
(b) Comparison of flow throughput for an aggressive setting of the parameter $p_{max} = 0.5$



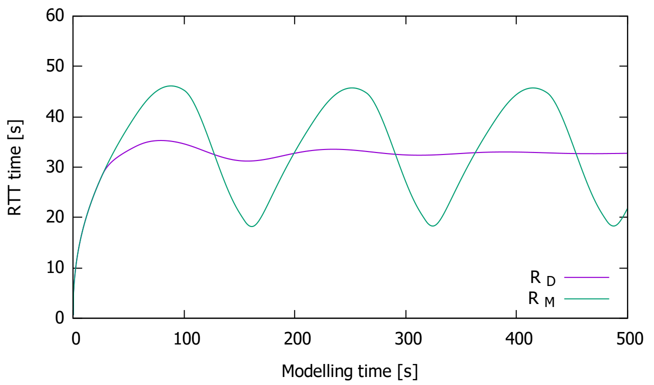
(c) Comparison of flow throughput for a mild setting of the parameter $p_{max} = 0.1$ in the range from 0.1 to 500 seconds



(d) Comparison of flow throughput for an aggressive setting of the parameter $p_{max} = 0.5$ in the range from 0.1 to 500 seconds



(e) Comparison of RTT times of the flow for a mild setting of the parameter $p_{max} = 0.1$



(f) Comparison of RTT times of the flow for an aggressive setting of the parameter $p_{max} = 0.5$

Fig. 2. Comaparison of numerical results of throughput and RTT times for a single flow and a single node operating according to the packet dropping (D) or packet marking (M) algorithm for the mild and aggressive setting of the p_{max} parameter. Individual categories of i flows are marked with colours.

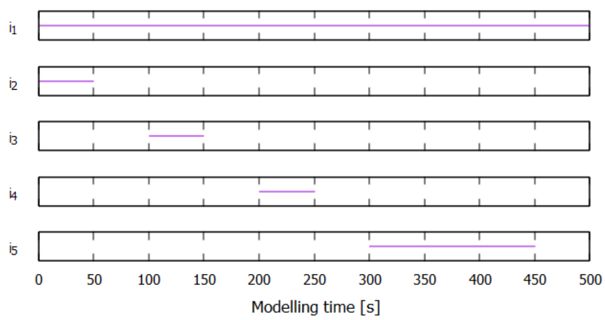


Fig. 3. Scheme of enabling and disabling individual i connection category over time

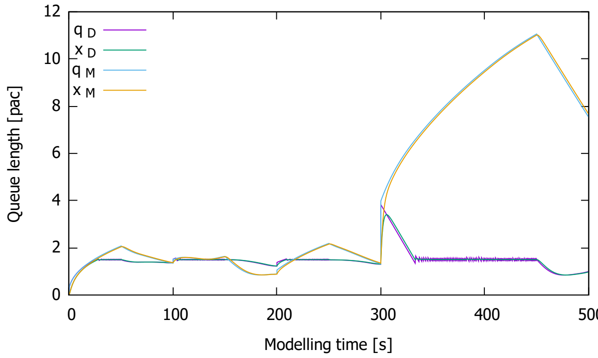


Fig. 4. Summary of the actual q_v and average x_v queue length of a router working according to the packet dropping (D) or marking (M) algorithm for a mild setting of the parameter $p_{max_v} = 0.1$

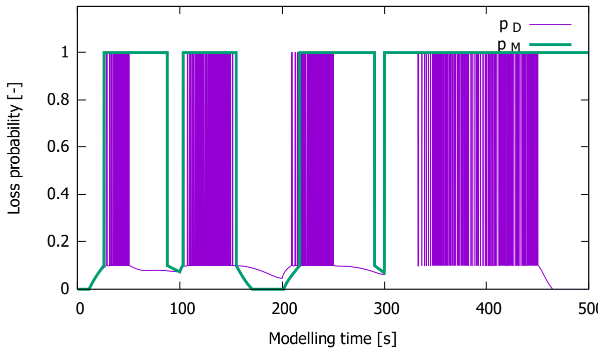


Fig. 5. Summary of the response probability of a router operating according to the packet dropping (D) or marking (M) algorithm for a mild parameter setting $p_{max_v} = 0.1$

of both models in the case of appearing and disappearing connections. The diagram of the impact time of individual flow categories on the node is shown in Fig. 3. Categories i_1-i_4 have single flows, while category i_5 consists of 16 identical flows. The parameters of the network router remained at the same level as before, but the analysis was limited only to the generally accepted probability $p_{max_v} = 0.1$.

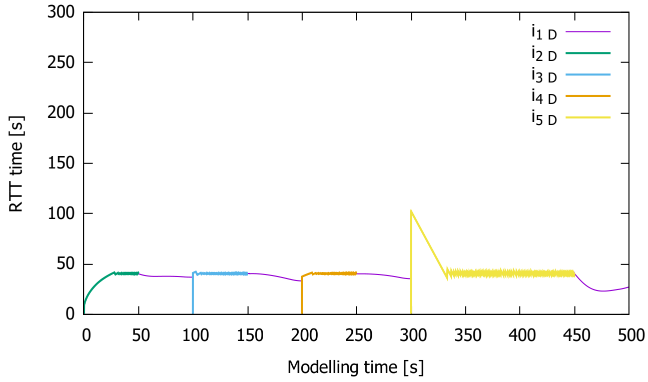
Significant differences in the RED algorithm's effect can be observed depending on the method used. In the case of the packet marking method, the impact of each attached or unattached connection in a node is significant, fig. 4. The most significant change occurs during heavy traffic ($t > 300$ s), where 16 flows are observed. At that moment, the average queue length q_v increases significantly. As it increases, the delay in informing the source about the network state increases, fig. 6b. Therefore, minimising the RTT time, which is crucial in wide-area networks, will not be possible in this case. However, the queue for model D, fig. 4 remains relatively at the same level. Even when there is a sudden increase in input traffic to a node, the algorithm quickly copes with this load by proactively discarding some input packets. It keeps the queue relatively low without triggering long-term loss generation, as with the M model, fig. 5.

The choice of method did not significantly affect the average size of the congestion window for short-lived connections i_2-i_4. However, it had an impact on long-term connections (i_1,i_5), for which it caused a smaller (fig. 6e) or greater (fig. 6f) reduction in the average size of active connection windows. In the case of the marking model, the reduction of the window size W_1 occurs even without the need to start a new connection ($t = 67$ s), the appearance of which only strengthened this effect and ultimately led to a decrease in the transmission quality i_1 - the sender had to limit the transmission down to the level of a single packet. When the i_5 category appears, the M model appropriately controls the behaviour of flows - the average size of their windows gradually increases. Unfortunately, this is related to a gigantic increase in the delay value, fig. 6b, synonymous with significantly underestimated losses visible at the sender, fig. 6d. The opposite situation occurs in the packet-dropping model. Apart from the temporary peak, it keeps the RTT parameter at the same level, fig. 6a, which leads to a fast pass of the information about the network status to the source. The sender, observing the intensification of losses, can react immediately. This approach results in much more minor fluctuations in the amplitude of the congestion window size for long-duration flows, as in the case of i_1, fig. 6e. Although the load added in 300 s ultimately causes an overload of the node (the total input traffic exceeds the capacity of the queue of the analysed router), and the flows of classes i_1 and i_5 are reduced to an initial value, this is the only such a drastic drop observed for the model with dropping packets. Model D tries to maintain the same transmission level, although it generates higher losses than model M, but these are selective preventive losses, fig. 6c.

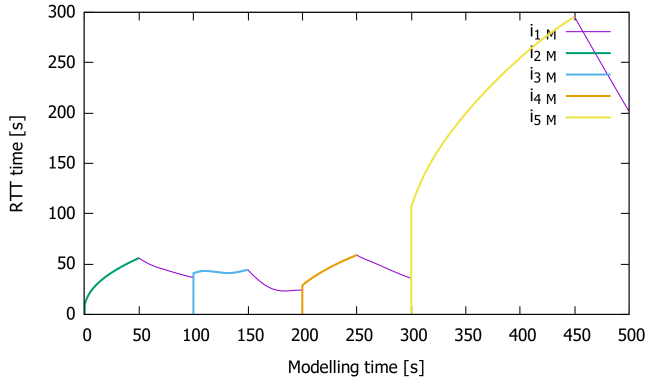
Figures 7a and 7b suggest that changing the model for the unloaded node does not affect the average throughput value. It is a false image due to the value jump above 25 pac/s when the i_5 category traffic appears. However, the values analysis showed that for the model working according to the D algorithm, on average, almost six times (and a maximum of 24 times) greater throughput is achieved than for the M model for long-term flow, fig. 7c and fig. 7d.

5. SUMMARY FOR THE CLASSIC MODEL

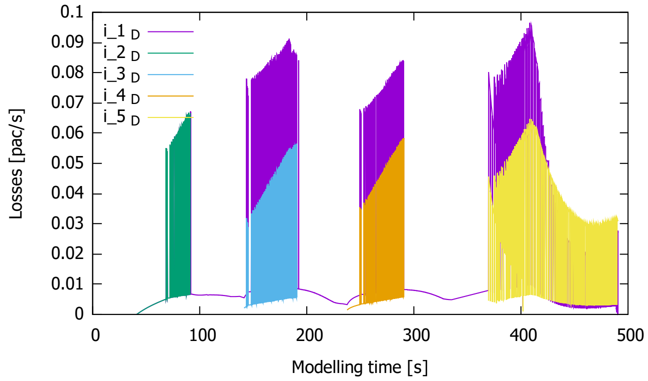
The RED algorithm, dropping packets after exceeding the maximum threshold, tries to reach a steady state whenever possible. It tries to keep losses constant while limiting oscillations and drastic drops in the average window size. It con-



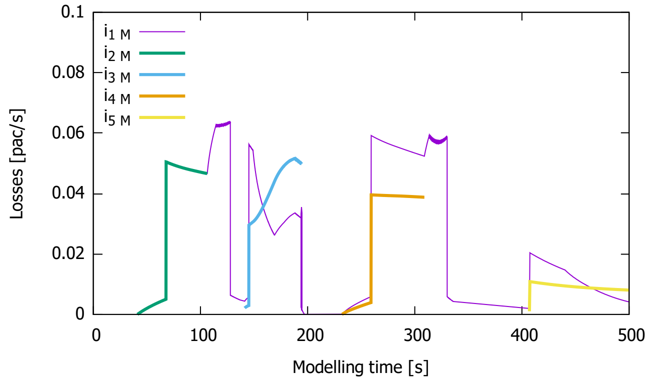
(a) Summary of RTT times for individual flow categories for model D



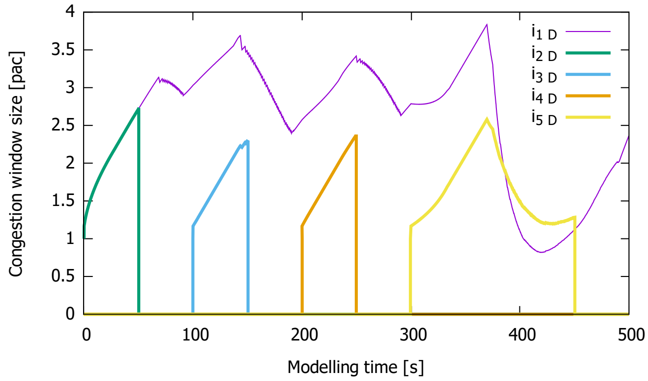
(b) Summary of RTT times for individual flow categories for model M



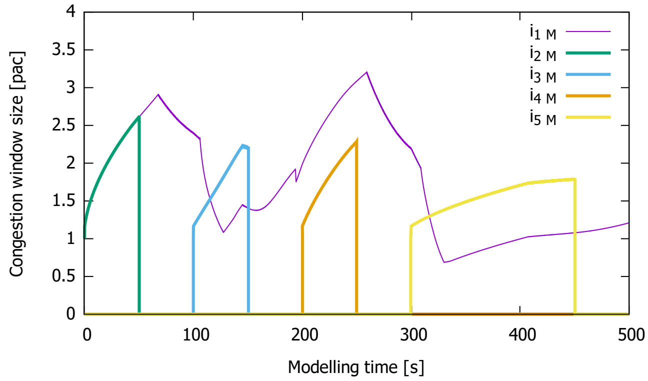
(c) Summary of the loss coefficient for individual flow categories for model D



(d) Summary of the loss coefficient for individual flow categories for model M

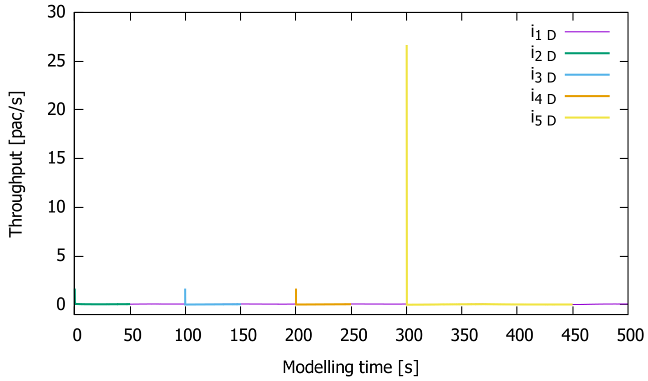


(e) Summary of congestion window sizes for individual flow categories for model D

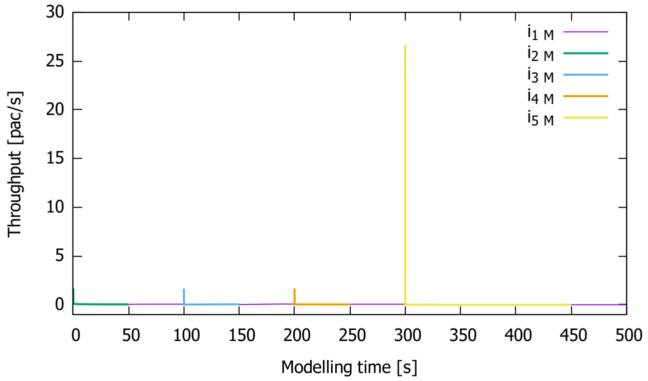


(f) Summary of congestion window sizes for individual flow categories for model M

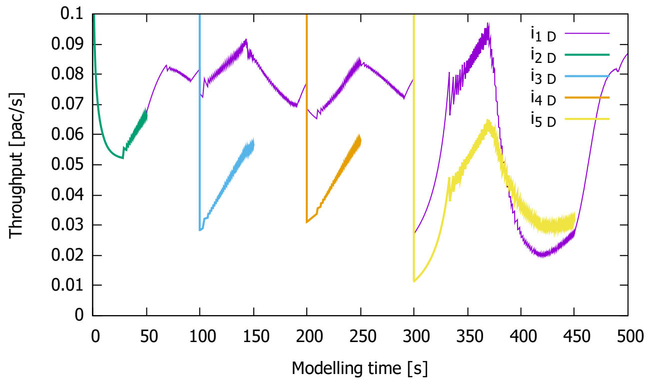
Fig. 6. Summary of numerical results of RTT times, loss coefficient and congestion window sizes for several flow categories and a single node operating according to the packet dropping (D) or marking (M) algorithm for a mild setting of the parameter $p_{max_v} = 0.1$



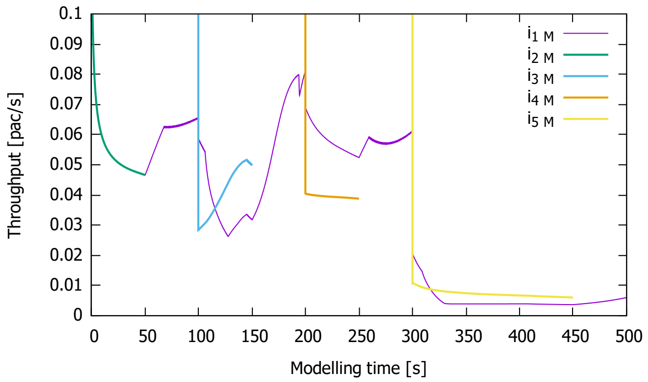
(a) Summary of flow throughputs for individual flow categories for model D



(b) Summary of flow throughputs for individual flow categories for model M

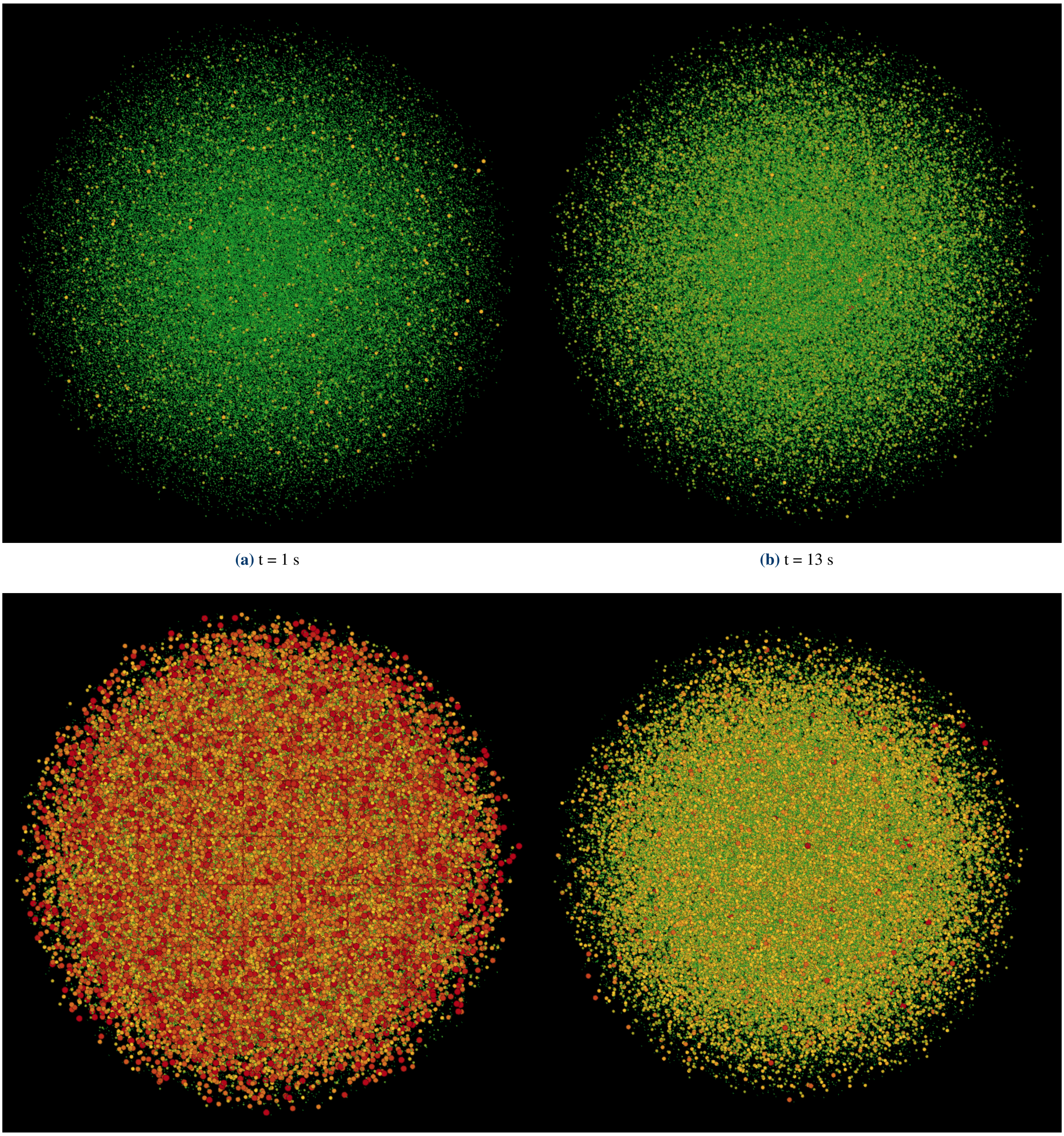


(c) Summary of flow throughputs for individual flow categories for model D zoomed in



(d) Summary of flow throughputs for individual flow categories for model M zoomed in

Fig. 7. Summary of numerical results of flow throughputs for several flow categories and a single node operating according to the packet dropping (D) or marking (M) algorithm for a mild setting of the parameter $p_{max_v} = 0.1$



(a) t = 1 s (b) t = 13 s

(c) t = 26 s (d) t = 50 s

Fig. 8. The load of the nodes in the modelled network for specified moments *t*

trols the use of the buffer, especially in moments of increased traffic, when it emphasises its preventive actions that are supposed to lead to solving the problem. It also has a slightly higher throughput than the packet marking model. The D-type model is a popular solution in numerous industrial devices, [5]. Therefore, using this model in the numerical analysis of the dynamics of queue changes in fluid-flow approximation seems justified. (6).

However, the formula (1) determining the RTT delay value, [24], requires a specification. The constant value of the propagation coefficient Tp_i has been replaced by the sum of the propagation values on individual links L_j between nodes, eq. (8), assuming that the number of edges in a given flow is E_i. This change is of key importance in the context of model analyses for wide-area networks where there are many flow routes.

$$R_i(t) = \sum_{j=1}^{N_i} \frac{q_j(t)}{C_j} + \sum_{j=1}^{E_i} L_j \quad (8)$$

6. NUMERICAL RESULTS FOR INTERNET SIZE TOPOLOGY

The described D model was used to analyse the actual internet topology [25, 26] of size 134 023 nodes through which 1 352 081 flows travelled, grouped into 50 000 classes. The results, presented in more detail in [27, 28], were visualised on a graph of the entire network, excluding the display of edges in the Gephi [29] tool. There is a colour division of routers in terms of the percentage of queue load:

- green - the queue is empty (0%),
- yellow - the queue is half full (50%),
- red - the queue is full (100%).
- intermediate colours - the queue is partially full (0-50% or 50-100%)

Figure 8 presents snapshots of the 'network life' at the most interesting moments, including a critical moment.

Such a network can also be analysed using a fluid-flow approximation regarding the global nature of changes, presenting individual or aggregated values across all nodes or connection categories over time, [30].

7. CONCLUSIONS

The RED algorithm detects potential network congestion and selects connections whose sources should be notified to reduce transmission by dropping the packet(s). The classical fluid-flow approximation model does not distinguish between methods of informing the source about congestion: packet dropping (D) and packet marking (M). However, when a packet is only marked, it is not physically removed from the node's queue, which means that even if the maximum RED threshold is exceeded, the queue may still grow. Conversely, when packets are dropped, the queue length does not increase. The article demonstrates that in the case of the packet-dropping model, both mild and aggressive settings positively reduce queue length oscillations and maintain transmission parameters, unlike the packet-marking model. Actual packet discarding reduces short- and long-term node congestion without excessively compromising transmission quality. The lack of distinction between these two methods in the classical model ultimately leads to erroneous numerical results.

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